5.4.2. Metoda parcijalne integracije za određene integrale

18. 12. 2020.

Fotografija: Roma Kaiuk

Parcijalna integracija za određene integrale

Neka su $u, v : I \subseteq \mathbb{R} \to \mathbb{R}$ derivabilne funkcije, i neka je $[a, b] \subseteq I$. Imamo

$$(uv)' = u'v + uv',$$

tj.

$$uv' = (uv)' - u'v.$$

Djelujemo li na ovu jednakost sa $\int_a^b$, dobivamo **formulu parcijalne integracije**

$$\int_a^b uv' = uv\Big|_a^b - \int_a^b u'v.$$

Popularno je (i korisno u računu) zapisivati je kratko i neprecizno u obliku

$$\boxed{\int_a^b u\,\mathrm{d}v = uv\Big|_a^b - \int_a^b v\,\mathrm{d}u.}$$

Primjer 1

Koristeći formulu parcijalne integracije

$$\boxed{\int_a^b u\,\mathrm{d}v = uv\Big|_a^b - \int_a^b v\,\mathrm{d}u,}$$

možemo računati

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} x\cos x\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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$$= \left(\frac{\sqrt{3}}{6} - \frac{\sqrt{2}}{8}\right)\pi + \frac{1-\sqrt{2}}{2}.$$

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Izračunajte integral $\int_2^3 x^2\, e^{2x}\,\mathrm{d}x$.

Rješenje. $\int_2^3 x^2\, e^{2x}\,\mathrm{d}x = \left[\begin{array}{cc} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{2x}\,\mathrm{d}x & v = \frac{1}{2}e^{2x} \end{array}\right]$

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Rješenje.

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$$= \left(\frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x}\right)\Big|_2^3$$

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$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 50(a)

Izračunajte integral $\displaystyle\int_2^3 x^2\, e^{2x}\, \mathrm{d}x$.

Rješenje. $\displaystyle\int_2^3 x^2\, e^{2x}\, \mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{2x}\,\mathrm{d}x & v = \frac{1}{2}e^{2x} \end{bmatrix}$

$$= \frac{1}{2}x^2 e^{2x}\Big|_2^3 - \int_2^3 x e^{2x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{2x}\,\mathrm{d}x & v = \frac{1}{2}e^{2x} \end{bmatrix}$$

$$= \frac{1}{2}x^2 e^{2x}\Big|_2^3 - \left(\frac{1}{2}x e^{2x}\Big|_2^3 - \frac{1}{2}\int_2^3 e^{2x}\,\mathrm{d}x\right)$$

$$= \left(\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x}\right)\Big|_2^3 = \frac{1}{2}e^{2x}\left(x^2 - x + \frac{1}{2}\right)\Big|_2^3$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

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$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 50(a)

Izračunajte integral $\displaystyle\int_2^3 x^2\, e^{2x}\,\mathrm{d}x$.

Rješenje.

$$\int_2^3 x^2\, e^{2x}\,\mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{2x}\,\mathrm{d}x & v = \frac{1}{2}e^{2x} \end{bmatrix}$$

$$= \frac{1}{2}x^2 e^{2x}\Big|_2^3 - \int_2^3 xe^{2x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{2x}\,\mathrm{d}x & v = \frac{1}{2}e^{2x} \end{bmatrix}$$

$$= \frac{1}{2}x^2 e^{2x}\Big|_2^3 - \left(\frac{1}{2}xe^{2x}\Big|_2^3 - \frac{1}{2}\int_2^3 e^{2x}\,\mathrm{d}x\right)$$

$$= \left(\frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x}\right)\Big|_2^3 = \frac{1}{2}e^{2x}\left(x^2 - x + \frac{1}{2}\right)\Big|_2^3$$

$$= \frac{1}{2}e^6\left(3^2 - 3 + \frac{1}{2}\right) - \frac{1}{2}e^4\left(2^2 - 2 + \frac{1}{2}\right)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 50(a)

Izračunajte integral $\int_2^3 x^2\, e^{2x}\, \mathrm{d}x$.

Rješenje. $\int_2^3 x^2\, e^{2x}\, \mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{2x}\,\mathrm{d}x & v = \frac{1}{2}e^{2x} \end{bmatrix}$

$$= \frac{1}{2}x^2 e^{2x}\Big|_2^3 - \int_2^3 xe^{2x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{2x}\,\mathrm{d}x & v = \frac{1}{2}e^{2x} \end{bmatrix}$$

$$= \frac{1}{2}x^2 e^{2x}\Big|_2^3 - \left(\frac{1}{2}xe^{2x}\Big|_2^3 - \frac{1}{2}\int_2^3 e^{2x}\,\mathrm{d}x\right)$$

$$= \left(\frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x}\right)\Big|_2^3 = \frac{1}{2}e^{2x}\left(x^2 - x + \frac{1}{2}\right)\Big|_2^3$$

$$= \frac{1}{2}e^6\left(3^2 - 3 + \frac{1}{2}\right) - \frac{1}{2}e^4\left(2^2 - 2 + \frac{1}{2}\right) = \frac{13}{4}e^6 - \frac{5}{4}e^4.$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix}$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \,\mathrm{d}x \\ \mathrm{d}v = e^x \,\mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \,\mathrm{d}x =$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \, \mathrm{d}x =$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \,\mathrm{d}x \\ \mathrm{d}v = e^x \,\mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \,\mathrm{d}x =$

$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \,\mathrm{d}x \\ \mathrm{d}v = e^x \,\mathrm{d}x & v = e^x \end{bmatrix}$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \,\mathrm{d}x \\ \mathrm{d}v = e^x \,\mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \,\mathrm{d}x =$

$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \,\mathrm{d}x \\ \mathrm{d}v = e^x \,\mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \,\mathrm{d}x \right)$$

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \, \mathrm{d}x =$

$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \, \mathrm{d}x \right)$$

$$= e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, dx$.

Rješenje. $$I = \int_{\frac{1}{2}}^{1} e^x \sin x \, dx = \begin{bmatrix} u = \sin x & du = \cos x \, dx \\ dv = e^x \, dx & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \, dx =$$

$$= \begin{bmatrix} u = \cos x & du = -\sin x \, dx \\ dv = e^x \, dx & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \, dx \right)$$

$$= e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \sin x \, dx = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - I,$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \,\mathrm{d}x \\ \mathrm{d}v = e^x \,\mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \,\mathrm{d}x =$

$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \,\mathrm{d}x \\ \mathrm{d}v = e^x \,\mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \,\mathrm{d}x \right)$$

$$= e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \sin x \,\mathrm{d}x = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - I,$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Rješenje.
$$I = \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \, \mathrm{d}x =$$
$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \, \mathrm{d}x \right)$$
$$= e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - I,$$
$$\Rightarrow \quad 2I = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1}$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \, \mathrm{d}x =$

$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \, \mathrm{d}x \right)$$

$$= e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - I,$$

$$\Rightarrow \quad 2I = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1}$$

$$\Rightarrow \quad I = \frac{1}{2} e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1}$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Rješenje. $I = \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \, \mathrm{d}x =$

$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \, \mathrm{d}x \right)$$

$$= e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - I,$$

$$\Rightarrow \quad 2I = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1}$$

$$\Rightarrow \quad I = \frac{1}{2} e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} = \frac{1}{2} \left(e \, (\sin 1 - \cos 1) - e^{\frac{1}{2}} \left(\sin \frac{1}{2} - \cos \frac{1}{2} \right) \right).$$

Zadatak 50(b)

Izračunajte integral $I := \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x$.

Oprez! Ako ovdje stavimo $u = e^x$, vratit ćemo se na početak!

Rješenje.

$$I = \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = \begin{bmatrix} u = \sin x & \mathrm{d}u = \cos x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \cos x \, \mathrm{d}x =$$

$$= \begin{bmatrix} u = \cos x & \mathrm{d}u = -\sin x \, \mathrm{d}x \\ \mathrm{d}v = e^x \, \mathrm{d}x & v = e^x \end{bmatrix} = e^x \sin x \Big|_{\frac{1}{2}}^{1} - \left(e^x \cos x \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x (-\sin x) \, \mathrm{d}x \right)$$

$$= e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} e^x \sin x \, \mathrm{d}x = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} - I,$$

$$\Rightarrow \quad 2I = e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1}$$

$$\Rightarrow \quad I = \frac{1}{2} e^x (\sin x - \cos x) \Big|_{\frac{1}{2}}^{1} = \frac{1}{2} \left(e (\sin 1 - \cos 1) - e^{\frac{1}{2}} \left(\sin \frac{1}{2} - \cos \frac{1}{2} \right) \right).$$